

Chapter 14: Circular Functions

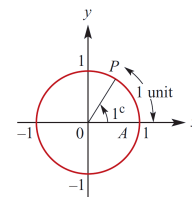
14A: Angles

Unit Circle: Circle of radius 1. $\Rightarrow x^2 + y^2 = 1$

Degree: Defined by dividing a circle into 360 equal parts.

Radian: Defined by the distance moved around the circumference of a unit circle.

1 unit = 1 radian



Degrees \rightarrow <u>Radians</u>	$\times \frac{\pi}{180^\circ}$	In <u>RAD.</u> (Setup 4) Value DRG \downarrow 1 =
Radians \rightarrow <u>Degrees</u>	$\times \frac{180^\circ}{\pi}$	In <u>DEG.</u> (Setup 3) Value DRG \downarrow 2 =

Practice Questions

Convert the following units:

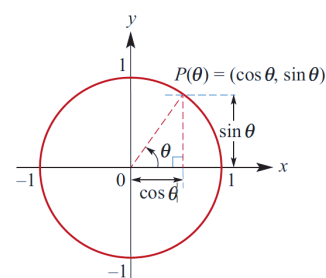
1. 144°	2. $\frac{5\pi}{9}$
3. 1.89°	4. 84.1°
5. $-\frac{\pi}{3}$	6. -540°

Questions: 1-7 Right half

14B: Sine & Cosine

x-coordinate of a point on the unit circle is given by $x = \cos \theta$.

y-coordinate of a point on the unit circle is given by $y = \sin \theta$.



Practice Questions

Find the sine and cosine of the following angles:

1. $\frac{3\pi}{2}$	2. 27π	3. 2.3°
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Questions: 1-3 RS (L,M,R- left hand, middle, right hand column. LS (etc) – left columns)

14C: Tangent

Distance (in y) along the line $x = 1$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

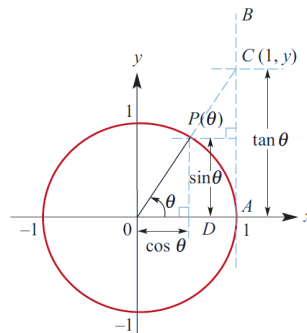
Undefined at $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$, etc.

Domain is $\mathbb{R} \setminus \{\theta: \cos\theta = 0\}$

Practice Questions

1) $\tan \pi$	2) $\tan(7\pi/2)$	3) $\tan(180^\circ)$
4) $\tan(-180^\circ)$	5) $\tan 1.6$	Remember to set calculator units!

Questions: 1bdef, 2bcd, 3RS



14D: Trig Ratios

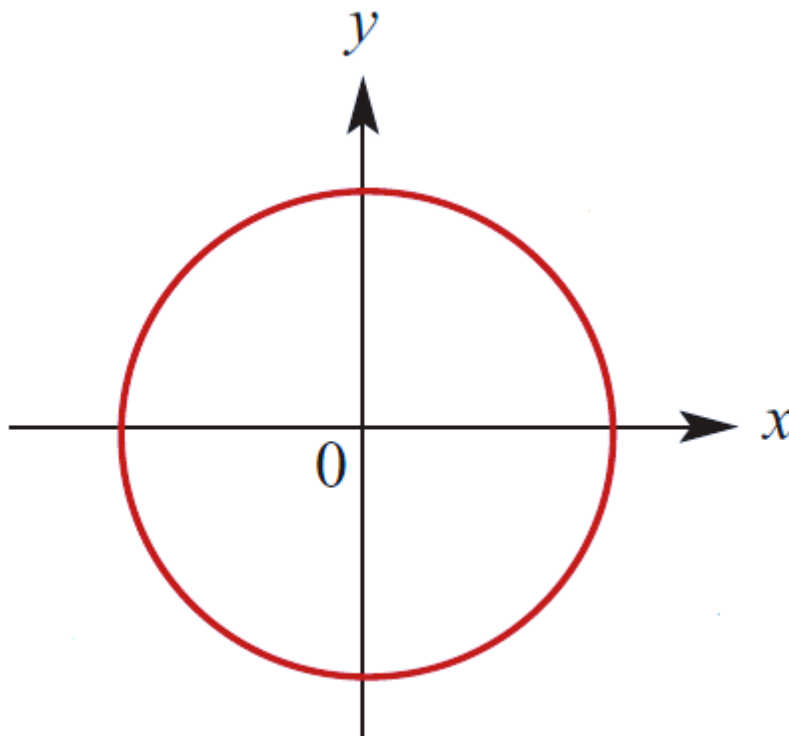
$\sin\theta = \frac{O}{H}$	$\cos\theta = \frac{A}{H}$	$\tan\theta = \frac{O}{A}$
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1. Determine the side lengths you have, to select appropriate ratio.
2. Sub in known & unknown values.
3. Rearrange if required, and solve.

Questions: All

14E: Symmetry – Part 1 (VERY IMPORTANT)

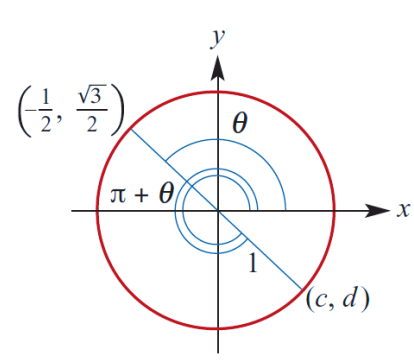
1. Label Quadrants
2. Add point P in quadrant 1.
3. Give coordinates.
4. Reflect P in y .
5. Give coordinates.
6. Reflect P in x .
7. Give coordinates.
8. Reflect point into Quad 3.
9. Give coordinates.
10. Add tangent line.
11. Find value of tangent to each point.
12. CAST to help remember which ratios are positive in which quadrant.



13. Negative values of angles

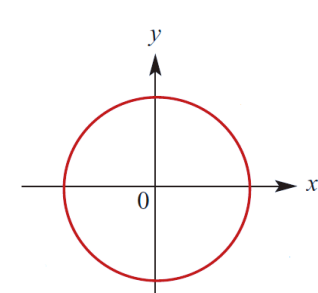
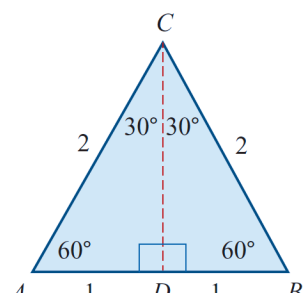
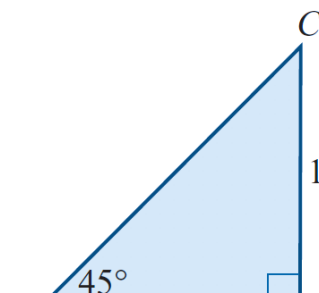
- $\cos(-\theta) =$
- $\sin(-\theta) =$
- $\tan(-\theta) =$

Practice Questions

1) If $\cos x = -\cos(\pi/6)$ and $\pi/2 < x < \pi$. Find the value of x .	2) For the diagram shown, write down the values of: a) $d = \sin(\pi + \theta)$ b) $c = \cos(\pi + \theta)$ c) $\tan((\pi + \theta))$ d) $\sin(2\pi - \theta)$ e) $\cos(2\pi - \theta)$
3) If $\sin x^\circ = -\sin 60^\circ$ and $-90^\circ < x^\circ < 0^\circ$, find the value of x° .	
4) If $\cos x^\circ = -\cos 60^\circ$ and $90^\circ < x^\circ < 180^\circ$, find the value of x° .	

Questions: ALL, except 2a, 4, 7cd.

14F: Exact Values – Very Useful

Unit Circle – $0^\circ, \pi/2$	Equilateral Triangle – $\pi/6, \pi/3$	Isosceles Triangle – $\pi/4$
		

Using the diagrams above complete the table:

	0°	$\frac{\pi}{6}, 30^\circ$	$\frac{\pi}{4}, 45^\circ$	$\frac{\pi}{3}, 60^\circ$	$\frac{\pi}{2}, 90^\circ$
sin					
cos					
tan					

Practice Questions

Without using a calculator find the exact values of:

1. $\sin 120^\circ$	2. $\cos 120^\circ$	3. $\tan 120^\circ$
4. $\sin 390^\circ$	5. $\cos 390^\circ$	6. $\tan 390^\circ$
7. $\sin \frac{2\pi}{3}$	8. $\cos \frac{3\pi}{4}$	9. $\tan \frac{5\pi}{6}$
10. $\sin -\frac{2\pi}{3}$	11. $\cos \frac{14\pi}{4}$	

Questions: 1M&RS, 2RS, 3RS.

14G: Graphs – Part 1

$$y = a \sin(nt) \text{ OR } y = a \cos(nt)$$

a – changes the	A = amplitude = distance from centre of wave to top (or bottom) Range =
n – changes the	P = Period = Tells how long one wavelength is. (n indicates how many waves are between $[0, 2\pi]$.)

Plotting Graphs: (This is how I do it – You may prefer to draw the axes in (4) and then place the graph (3).)

- 1) Identify Shape
- 2) Identify A, P
- 3) Draw cycle(s). Look to domain and Period to indicate how many times.
- 4) Add in x & y axes.
- 5) Label known key points (y-intercept, start/end, centre.)
- 6) Label other key points. (x & y minimums & maximums, start/end.)

Practice Questions

Sketch the following graphs. Show one complete cycle, stating amplitude and period.

1) $y = 3 \sin 2x$	2) $y = -3 \cos \frac{\theta}{2}$
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Questions: 1, 3RS, 4, 5, 8

14H: Solving Trig Equations

- 1) Rearrange equation into the form $\sin(\theta_{\text{ref}}) = a$ or $\cos(\theta_{\text{ref}}) = a$
- 2) Solve for θ_{ref} between 0 and π .
- 3) Use symmetry or unit circle to find other required values of θ_{ref} .
- 4) Solve for θ . (May need to adjust number of θ_{ref} .)

Practice Questions

Solve the following for x .

1) $2 \cos x = \sqrt{3}$ (Between 0 and 2π)	2) $\cos x = -\frac{1}{\sqrt{2}}$ (Between 0 and 4π)
3) $\sin 2\theta = -\frac{1}{2}$ (Between $-\pi$ and π)	

Questions: 1-7 RS

14I: Graphs – Part 2

$$y = a \sin n(t \pm \epsilon) \text{ OR } y = a \cos n(t \pm \epsilon)$$

ϵ – Shifts the graph	$+\epsilon$ moves the graph $-\epsilon$ moves the graph
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Plotting Graphs: (This is how I do it – You may prefer to draw the axes in (4) and then place the graph (3).)

- 1) Identify Shape
- 2) Identify A, P, & ϵ
- 3) Draw cycle(s). Look to domain and Period to indicate how many times.
- 4) Add in x & y axes. (Extend graph to fit domain.)
- 5) Label known key points (y-intercept, start/end, centre.)
- 6) Find & label x-intercepts
- 7) Label other key points. (x & y minimums & maximums, start/end.)

Practice Questions

Sketch the following graphs. Show one complete cycle, stating amplitude and period. Find x& y intercepts

1) $y = \sqrt{3} \sin 2(\theta - \frac{\pi}{2})$

2) $y = 2 \cos 3(\theta + \frac{\pi}{4})$

Questions: 1 (not d,f) – x-ints not required. 2-4 x-ints reqd.

14J: Graphs – Part 3

$$y = a \sin n(t \pm \varepsilon) \pm b \text{ OR } y = a \cos n(t \pm \varepsilon) \pm b$$

b – Shifts the graph

$+b$ moves the graph

$-b$ moves the graph

Plotting Graphs: (This is how I do it – You may prefer to draw the axes in (4) and then place the graph (3).)

- 1) Identify Shape
- 2) Identify A, P, ε & **b** .
- 3) Draw cycle(s). Look to domain and Period to indicate how many times.
- 4) Add in x & y axes. (Extend graph to fit domain.)
- 5) Label known key points (y-intercept, start/end, centre.)
- 6) Find & label x-intercepts
- 7) Label other key points. (x & y minimums & maximums, start/end.)

Practice Questions

1) $y = 2 \cos 2 \left(x - \frac{\pi}{3} \right) - 1, [-\pi, 2\pi]$

Questions: All. (Except 2e.)

14K: Symmetry – Part 2

Complementary Relationships		Pythagorean Identity
$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$	$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$	$\cos^2 \theta + \sin^2 \theta = 1$
$\sin\left(\frac{\pi}{2} + \theta\right) = \cos \theta$	$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta$	

Practice Questions

1) If $\sin x = 0.3$, $\cos \alpha = 0.6$ and $\tan \theta = 0.7$, find the values of:		
b) $\sin\left(\frac{\pi}{2} + \alpha\right)$	f) $\tan\left(\frac{\pi}{2} - \theta\right)$	j) $\cos\left(\frac{3\pi}{2} - x\right)$
2) Given that $0 < \theta < \frac{\pi}{2}$ and $\sin \theta = \cos\left(\frac{\pi}{6}\right)$, find θ .		3) Given that $\sin x = \frac{5}{13}$ and $\frac{\pi}{2} < x < \pi$, find $\cos x$ and $\tan x$.

Questions: 1RS, 2cd, 3, 5.

14L: Tangent Graph

Similar method to 14J.

Note that:

- 1) The period of a tan graph is π .
- 2) The range of tan is \mathbb{R} .
- 3) The equations of the asymptotes (for a graph not translated) are of the form $x = \frac{(2k+1)\pi}{2n}$
- 4) The x-axis intercepts are $x = \frac{k\pi}{n}$ (for a graph not translated).

Practice Questions

1) Sketch the graph of $y = -3 \tan(2x)$ for $x \in [-\pi, \pi]$.
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2) Solve $2 \tan(2x) = 2\sqrt{3}$ for $x \in [-\pi, \pi]$.

Questions: 1 (period only), 2RS, 3R, 4RS

14M: Numerical Methods – CAS

Plug into CAS.

Practice Questions

In class.

Questions: Some – as a class.

14N: General Solutions

Using n, instead of actually calculating each value. Reflects what CAS results look like.

Practice Questions

In class.

Questions: Some – as a class.

14O: Applications

Working through some of these will make up the assessment for this topic. I'll go through 2, the rest are to be submitted with clear calculations and answers given in a sentence that states the question. More info to come later.